Int J Theor Phys (2009) 48: 1405–1414
DOI 10.1007/s10773-008-9910-z

Effect of Cavity Decay on the Coherent Control of Atomic Tunneling

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Received: 19 August 2008 / Accepted: 25 November 2008 / Published online: 2 December 2008

Abstract In this paper we study the influence of cavity decay on the atomic tunneling and entanglement dynamics in a cavity QED system. The system consists of an atom in a double-well potential and a cavity. The results show that the cavity decay affects significantly the tunneling and the entanglement dynamics. The tunneling behaves as a damping-oscillating function of time in this case, while the entanglement shared between the internal and external degree of freedom of the atom exhibits the so-called entanglement sudden death (ESD).

Keywords Coherent control · Tunneling · Entanglement · Sudden death

1 Introduction

The tunneling effect and entanglement are considered as two features that distinguish the quantum and classical world. Since the quantum tunneling for intramolecular rearrangements in pyramidal molecules was testified in 1927 [1] a lot of work has been done to study the tunneling [2–12]. In a double-well potential, taking the atomic internal degrees of freedom and atom-cavity coupling into account, the tunneling has been predicted very recently in [13]. It has been shown that when taking into account the interaction between the internal and external degrees of freedom the tunneling process becomes quasiperiodic. The periodic tunneling motion is modulated by the Rabi oscillations between states with a photon in the cavity and states with an excitation in the atom.

On the aspect of entanglement [14], a recent work of Yu and Eberly [15, 16] predicted that two noninteracting atoms initially entangled and coupled to two separate environments can become completely disentangled in a finite time. This is the so-called entanglement sudden death (ESD). Putting two atoms inside the same cavity, the ESD has been addressed by Tanaś in [17].

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In this paper we consider the same system as in [13]. Moreover we add the cavity decay and the spontaneous emission of the atom to the system. We show that the tunneling exhibits the property of quasiperiodic with damping amplitude, at the same time the so-called ESD appears. In the non-resonant case, the evolution of average position oscillates with time t after a period of initial damping.

The structure of this paper is organized as follows. In Sect. 2, we introduce the model. In Sect. 3 we discuss the dynamics of tunneling and entanglement for this system. In Sect. 4 we present conclusions and discussions.

2 Model

We consider the same model as given by Martin and Braun [13] that a two-level atom is confined in a cavity. The atom interacts with a standing wave cavity mode. The cavity is assumed as a symmetric double well potential $V(x)$ along the x-direction putted on the atom. The atom is supposed to be bound in the y–z plane at the equilibrium position $y = z = 0$. Such a model may be described by the following Hamiltonian,

$$H = H_A + H_F + H_{AF}, \tag{1}$$

where H_A is the Hamiltonian of the atom with $H_A = H_A^{\mathrm{ex}} + H_A^{\mathrm{in}}$, H_F is the Hamiltonian of the free field and H_{AF} denotes the atom-field interaction. We have [13]

$$\begin{aligned} H_A^{\mathrm{ex}} &= \tfrac{p_x^2}{2m} + V(x), \\ H_A^{\mathrm{in}} &= \tfrac{\hbar\omega_0}{2}\sigma_z^{\mathrm{in}}, \\ H_F &= \hbar\omega a^\dagger a, \\ H_{AF} &= -\mathbf{d}\cdot\mathbf{E}, \end{aligned} \tag{2}$$

where H_A^{ex} and H_A^{in} denote the Hamiltonian of the external and the internal degrees of freedom of the atom. $|e\rangle$ and $|g\rangle$ are the excited state and the ground state for H_A^{in} with energy $\pm\frac{\hbar\omega_0}{2}$ respectively. m denotes the atomic mass, p_x is the momentum along the x-axis. $\mathbf{d}$ denotes the atomic dipole, the electric field operator [13]

$$\mathbf{E} = E_\omega \vec{\varepsilon}(a + a^\dagger)\sin(k(x - x_0)), \tag{3}$$

with $E_\omega = \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}}$, V is the electromagnetic mode volume, ε_0 is the permittivity of free space, $\vec{\varepsilon}$ is the electric field polarization vector, x_0 denotes the abscissa at the left cavity mirror ($x_0 < 0$).

In the following we will adopt the two-level approximation of the motion in the external potential, namely consider only the two lowest energy levels of the Hamiltonian H_A^{ex}. Within this approximation, H_A^{ex} becomes

$$H_A^{\mathrm{ex}} = \frac{\hbar\Delta}{2}\sigma_z^{\mathrm{ex}}. \tag{4}$$

The operators σ_i^{in} (resp. σ_i^{ex}) are the Pauli operators for $i = x, y, z$ in the bases $\{|e\rangle, |g\rangle\}$ (resp. $|+\rangle, |-\rangle$). The states that the atom are mainly confined in the left and right well can be written as, $|L\rangle = (|+\rangle - |-\rangle)/\sqrt{2}$, $|R\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$. Δ denotes the tunnel splitting (the symmetric $|-\rangle$ and antisymmetric $|+\rangle$ states) of the double-well potential. The position operator for the center-of-mass position of the atom localized in the right well is given by $x = b/2\,\sigma_x^{\mathrm{ex}}$ for $b/2 = \langle +|x|-\rangle$, $\sigma_x^{\mathrm{ex}} = |R\rangle\langle R| - |L\rangle\langle L|$. Then the interaction Hamiltonian H_{AF} can be written as

$$H_{AF} = -\hbar g(a + a^\dagger)[\sin\chi\cos\kappa\sigma_x^{\mathrm{in}} - \cos\chi\sin\kappa\sigma_x^{\mathrm{ex}}\sigma_x^{\mathrm{in}}], \tag{5}$$

where $\chi = kx_0$, $\kappa = kb/2$, and $g = -\langle e|\mathbf{d}|g\rangle\vec{\varepsilon}E_\omega/\hbar$ is the atom-field coupling strength.

The two-level approximation is valid when the recoil energy $\hbar\omega_{\mathrm{recoil}}$ is much smaller than the difference in energy to the next highest vibrational level $\hbar\widetilde{\Delta}$ in the external potential [18]. Numerical simulations show that for $\kappa \sim 1$ the two-level approximation can still work very well.

For $\delta, \Delta \ll \omega, \omega_0$, a rotating wave approximation is a good approximation, leading to eliminating the energy non-conserving terms $a\sigma_\pm^{\mathrm{ex}}\sigma_-^{\mathrm{in}}$ and $a^\dagger\sigma_\pm^{\mathrm{ex}}\sigma_+^{\mathrm{in}}$. Within this approximation, the Hamiltonian given in (1) can be written as [13]

$$\begin{aligned} H = {} & \frac{\hbar\Delta}{2}\sigma_z^{\mathrm{ex}} + \frac{\hbar\omega_0}{2}\sigma_z^{\mathrm{in}} + \hbar\omega a^\dagger a \\ & + \hbar g\cos\chi\sin\kappa(a\sigma_+^{\mathrm{ex}}\sigma_+^{\mathrm{in}} + a\sigma_-^{\mathrm{ex}}\sigma_+^{\mathrm{in}} + h.c.) \\ & - \hbar g\sin\chi\cos\kappa(a\sigma_+^{\mathrm{in}} + a^\dagger\sigma_-^{\mathrm{in}}), \end{aligned} \tag{6}$$

where $\sigma_+^{\mathrm{in}} = |e\rangle\langle g|$, $\sigma_-^{\mathrm{in}} = \sigma_+^{\mathrm{in}\dagger}$, $\sigma_+^{\mathrm{ex}} = |+\rangle\langle -|$ and $\sigma_-^{\mathrm{ex}} = \sigma_+^{\mathrm{ex}\dagger}$. k and ω are wave number and frequency of the standing wave, respectively. And a ($a^\dagger$) is the annihilation (creation) operator of the field. $\delta = \omega - \omega_0$ is the detuning between the cavity field and the atomic transition frequencies.

The global state of the atom-field system is denoted by $|n, i, j\rangle \equiv |n\rangle \otimes |i\rangle \otimes |j\rangle$, where $|n\rangle$ are the eigenstates for cavity field, $|i\rangle \in \{|-\rangle, |+\rangle\}$ are the external motional states and $|j\rangle \in \{|g\rangle, |e\rangle\}$ are the internal states. $a^\dagger a + \sigma_+^{\mathrm{in}}\sigma_-^{\mathrm{in}}$ gives the total excitation number N.

3 Internal and External Dynamics

Taking the cavity decay into account, the time evolution of the system is governed by the Markovian master equation [19, 20]

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H, \rho] + \Gamma\left\{a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right\}, \tag{7}$$

where H is the total Hamiltonian for the system. ρ is the density matrix of the system. The average position for the atom in the double well potential is given by [13]

$$\langle x\rangle = \frac{b}{2}\mathrm{Tr}_{\mathrm{ex}}(\rho^{\mathrm{ex}}\sigma_x^{\mathrm{ex}}) = \frac{b}{2}(1 - 2\rho_{LL}), \tag{8}$$

ρ^{ex} is the reduced density matrix by tracing out the internal degrees of freedom and the field, $\rho_{LL} = \langle L|\rho^{\mathrm{ex}}|L\rangle$. Similarly, the internal atomic state ρ^{in} is the reduced density matrix by tracing out the external degrees of freedom and the field. $\rho_{ee} = \langle e|\rho^{\mathrm{in}}|e\rangle$ is the probability of the atom in the excited state. The last term of the master equation presents the decay of the cavity, Γ is the cavity decay coefficient.

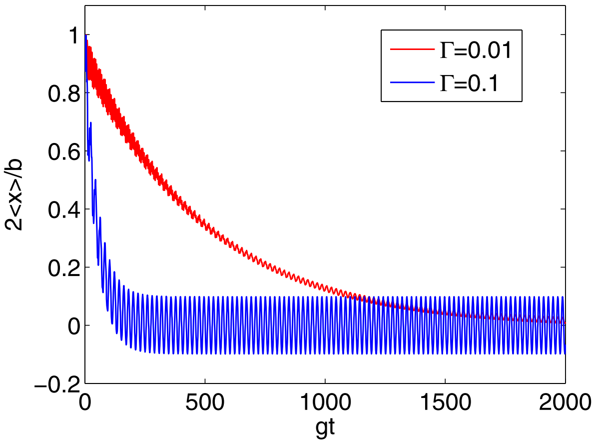

Fig. 1 (Color online) The average position of the atom in the double-well as a function of time t with the parameter $N=1$, $\Delta/g=0.32$, $\Gamma/g=0.004$ (*red curve*), $\Gamma/g=0.04$ (*blue curve*), $\kappa=\pi/4$, and for an excited atom located initially in the right well

Next we will study the tunneling of the system in the case of resonant atom-field interaction ($\delta=\omega-\omega_0=0$) and the case of non-resonant ($\delta\neq 0$) coupling. The entanglement between the internal and external degrees of freedom is also studied for the first case.

3.1 Resonant Atom-Field Interaction

3.1.1 Atom Tunneling

In this section we consider the system with resonant ($\delta=0$) atom-field couplings. We assume the initial state of the system is prepared in the state $|N-1,R,e\rangle$, namely the atom is in its excited state and in the right well, while the cavity field is in a Fock state $|N-1\rangle$.

We have performed extensive numerical simulations for the dynamics of the system, selected numerical results are presented in the following. In Fig. 1, we show the average position of the atom as a function of time in the case of $\Delta/g\ll 1$, for $N=1$, $\Delta/g=0.32$, $\Gamma/g=0.004$ (red curve), $\Gamma/g=0.04$ (blue curve) and $\kappa=\pi/4$. From this figure, we can find that the average position of the atom decays and averagely approaches to zero. The amplitude of oscillations depends on Γ when other parameters are fixed, the smaller the Γ is, the slower the decay is, and the larger the Γ, the larger the oscillating amplitude. This feature can be understood as the competition between the tunneling (proportional to Δ), excitation (governed by $a\sigma_-^{\text{ex}}\sigma_+^{\text{in}}$ $a^\dagger\sigma_+^{ex}\sigma_-^{in}$) and the cavity decay characterized by Γ. When excitation terms are ignored, the oscillations would not take place. Further simulations show that undamped tunneling with amplitude 1 happens when $\kappa=0$. It is obvious that in the Hamiltonian (6) when $\kappa=0$, there is no interaction between internal and external degrees of freedom, consequently the coupling of the atom to the environment does not affect the tunneling. For the case of $\kappa=\pi/2$ we have a result similar to the case with $\kappa=\pi/4$. The only difference is in the case with $\kappa=\pi/2$ the oscillations disappear, this can be understood by examining the Hamiltonian (6), where the coupling of the cavity field to the atomic internal degree of freedom is zero.

To simulate the dynamics in regime $\Delta\sim g$, we choose the cavity field initially in a coherent state

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{9}$$

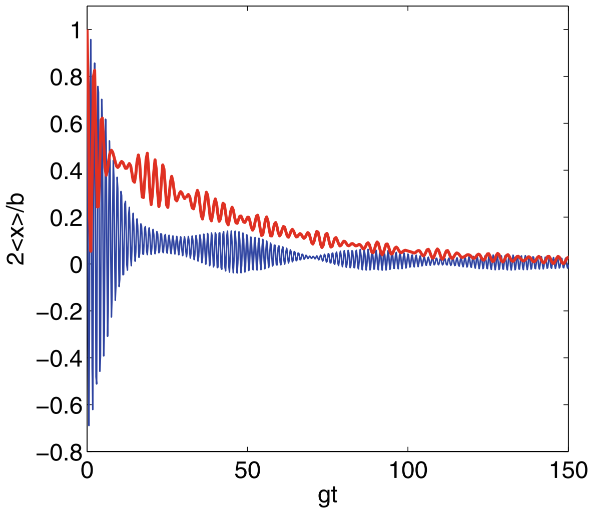

Fig. 2 (Color online) The time evolution of the average position for the parameters $N = 1$, $\kappa = \frac{\pi}{4}$, $\Delta/g = 5$ (*blue curve*), $\Delta/g = 2$ (*red curve*) and $\Gamma/g = 0.05$ and a coherent state with $|\alpha|^2 = 2$, both the curves correspond an excited atom initially in the right well

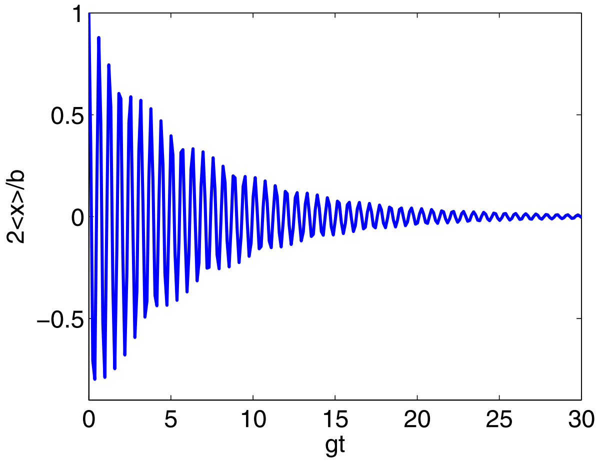

Fig. 3 (Color online) The average position of the atom in the double well as a function of time t with the parameters $N = 1$, $\Delta/g = 10$, $\Gamma/g = 2$, $\kappa = \frac{\pi}{4}$ for an excited atom located initially in the right well

when $|\alpha|^2$ equals to the mean photon number $\langle n \rangle$ of the cavity field. The time evolution of average position is exhibited in the Fig. 2. It is observed that for a coherent state, the average position of the atom shows the character of collapses and revivals, and the oscillation amplitude is related to the tunnel splitting Δ. As the Δ decreases the oscillation amplitude decreasing. Nonzero Γ leads the average position to decay to 0.

For large tunneling, i.e. $\Delta/g \gg 1$, ρ_{LL} behaves as a sine function of time when the cavity decay Γ is zero, namely $\rho_{LL} \approx \sin^2(\Delta t/2)$ [13]. The average position of the atom as a function of time in the double-well with the cavity decay is shown in the Fig. 3. The oscillation amplitude is decreased due to the cavity loss Γ. As the time evolution the average position asymptotically approaches to 0.

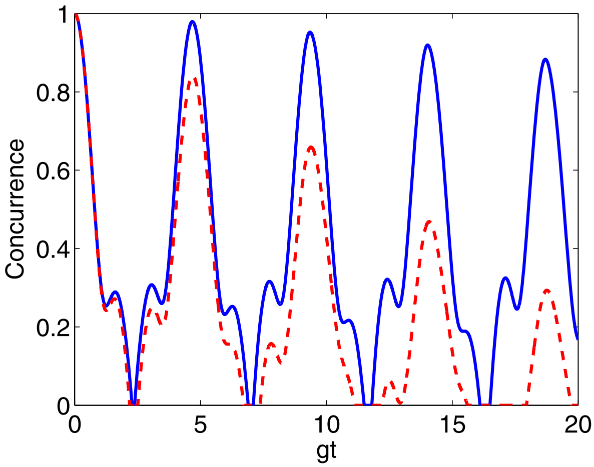

Fig. 4 (Color online) The time evolution of the concurrence for different Γ with the atom initially in the entangled state $|\Psi_{\text{atom}}\rangle = \cos\alpha|\uparrow\uparrow\rangle + \sin\alpha|\downarrow\downarrow\rangle$, $\alpha = \frac{\pi}{4}$, $\Delta/g = 1$, $\Gamma/g = 0.001$ (*blue solid curve*), $\Gamma/g = 0.1$ (*red dash curve*)

3.1.2 Entanglement

Now a numerical analysis of the dynamical sensitivity and the entanglement decay are in order. Wootters concurrence [21] is chosen as the measure of entanglement between the internal and the external degrees of freedom. In Fig. 4, the time evolution of the concurrence is plotted when the atom is initially in the maximal entangled state $|\Psi_{\text{atom}}\rangle = 1/\sqrt{2}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$. In this section $|\uparrow\rangle$ is introduced for denoting the states $|e\rangle$ and $|+\rangle$, at the same time $|\downarrow\rangle$ denoting the states $|g\rangle$ and $|-\rangle$. An important character of this figure is the collapses and revivals of the concurrence, namely, the entanglement can fall to zero abruptly, and before the entanglement recovers it remains to be zero for a period of time. This phenomenon, the entanglement becomes completely zero for a finite-time, was first found by Yu et al. [15, 16]. The length of the time interval for the zero entanglement, obviously, is dependent on the ratio of the decay coefficient to the coupling strength Γ/g. The larger the Γ/g is, the longer the time for the system remaining in separable states.

Another remarkable character that can be observed in Fig. 4 is the decay of the peak of the revival. As the Γ/g increases, the decay becomes faster. It is believed that the decay comes from the non-zero Γ.

Now we try to understand the ESD in our system. To this end, we assume an initial state represented by a density matrix,

$$\rho(0) = \begin{pmatrix} \frac{1}{10} & 0 & 0 & 0 \\ 0 & \frac{2}{5} & \frac{1}{4} + \frac{i}{4} & 0 \\ 0 & \frac{1}{4} - \frac{i}{4} & \frac{2}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{10} \end{pmatrix}. \tag{10}$$

We can numerically calculate the density matrix at time t $\rho(t)$, and then extract the element $|\rho_{23}(t)|$ of the density matrix, where $\rho_{23}(t)$ was defined by $\rho_{23}(t) = \langle +, g|\rho(t)|-, e\rangle$. Earlier studies on the entanglement show that this element of the density matrix is closely related to the entanglement, so we show $|\rho_{23}(t)|$ as a function of time in Fig. 5, for $N = 1$, $\Delta/g = 1$, $\kappa = \pi/4$, $\Gamma/g = 0.01$. Two observations can be made from the figure. (1) The larger the $|\rho_{23}|$ is, the stronger the entanglement in the system, (2) the entanglement sudden death

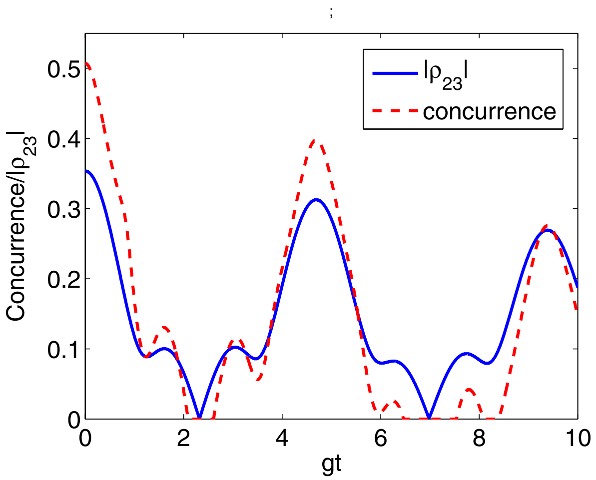

Fig. 5 (Color online) The time evolution of the concurrence (*red dash curve*) and $|\rho_{23}|$ (*blue solid curve*) for the parameter $N = 1$, $\Delta/g = 1$, $\kappa = \pi/4$, $\Gamma/g = 0.01$ with the atom in the initial state with density matrix ρ_0

happens when $|\rho_{23}(t)|$ is smaller than a specific value. Although the relations between the entanglement and $|\rho_{23}(t)|$ are evident, it is however a specific case and hence this relation can merely be generalized for all coupled systems, because the initial state and dynamics are not general. This requires us to analyze the ESD case by case. Recently, a great deal of works are dedicated to understanding the ESD from energy exchange between the system and its environment [22–25]. And the interpretation that the phenomena of ESD and disentanglement stem from the dissipative terms in the Hamiltonian has been provided based on the open system analysis (Tavis-Cumming model and dephasing model), the analysis for closed systems was given by Cui et al. [26]. It maybe provides another way to understand the dynamics of entanglement. Other works about the ESD see e.g. [27–39].

3.2 Non-Resonant Atom-Field Interaction

In this section we discuss the case of non-resonant atom-field interaction ($\delta \neq 0$). The damping of atomic tunneling in the non-resonant case was considered by Vukics et al. [40] and Maschler et al. [41]. In the Fig. 6 we show both influences of spontaneous emission (dash-dot, red curve) and the cavity decay (dash, blue curve) on the system. The spontaneous emission of the system has been discussed by Braun and Martin [18]. For the spontaneous emission and the cavity decay, the evolution of the average position with time t for the parameters $\Delta/g = \delta/g = 1$, $\gamma/g = 0.5$ (γ is the spontaneous emission coefficient), $\Gamma/g = 0.5$, $\kappa = \pi/4$ and $N = 1$ are shown in Fig. 6. For the case of spontaneous emission the tunneling exists and the evolution of the average position oscillates with time t after a period of initial damping. This result is the same as [18] when $\gamma \leq \Delta$. For the cavity decay we get the similar oscillations with spontaneous emission. It is can be said that the spontaneous emission and the cavity decay have the similar effect on the system. But when the spontaneous emission coefficient and the cavity decay coefficient are the same, namely $\gamma = \Gamma = 0.05$, the cavity decay gives a relatively stronger effect on the system. From the figure we can see that when the oscillation is steady the amplitude of spontaneous emission (dash-dot, red curve) is larger than the amplitude of cavity decay (dash, blue curve). When $\gamma \gg \Delta$ for the parameter $\Delta/g = 1$, $\delta/g = 1$ and $\gamma/g = 10$ we got the result that the tunneling motion is not affected by the decoherence for the case of spontaneous emission because the photon is emitted before the atom starts its tunneling.

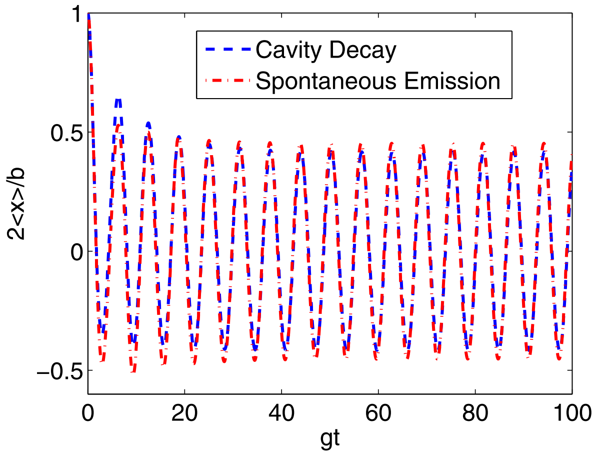

Fig. 6 (Color online) The average position of the atom as a function of time in the double well for the parameters $\Delta/g=\delta/g=1, \gamma/g=0.5$ (*dash-dot, red curve*), $\Gamma/g=0.5$ (*dash, blue curve*), $\kappa=\frac{\pi}{4}$, $N=1$. The excited atom initially located in the right well

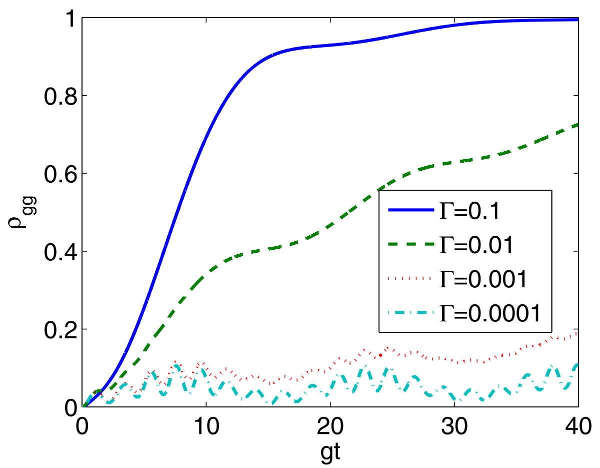

Fig. 7 (Color online) The evolution of the density matrix elements ρ_{gg} with time t for an excited atom initially located in the right well with parameters $N=1, \kappa=\frac{\pi}{4}, \Delta/g=0.336$, $\delta/g=3$, $g=0.01$, and $\Gamma/g=10$ (solid curve), $\Gamma/g=1$ (*dash curve*), $\Gamma/g=0.1$ (*dot curve*), $\Gamma/g=0.01$ (*dash-dot curve*)

In Fig. 7, the evolution of the density matrix element ρ_{gg} with time t for the parameters $N=1, \kappa=\frac{\pi}{4}, \Delta/g=0.336, \delta/g=3$, and $\Gamma/g=10, \Gamma/g=1, \Gamma/g=0.1$ and $\Gamma/g=0.01$ are exhibited. Initially the atom is excited and located in the right well. It shows that when $\Gamma/g=10$, for the reason of the cavity decay, the density matrix element ρ_{gg} increases to 1 asymptotically which are different to the result given by Martin [13] that the density matrix element ρ_{gg} oscillates. It is obviously that if the evolution time is long enough the probability to find the atom in the excited state is 0, namely, the atom decays into the ground state. The figure also shows that the smaller the Γ is the slower the ρ_{gg} approaches to 1. Further simulation shows that as Γ decreased ρ_{gg} exhibits the property of oscillation. As $\Gamma\to 0$ the results approach to the results reported in [13].

4 Conclusions

In this paper, we have mainly studied the effect of cavity decay on the tunneling and the entanglement evolution of a two-level atom in a double-well potential. The effect of spontaneous emission is also discussed briefly. The possible realization of this system has been presented by Braun and Martin [18]. Some interesting phenomena have been predicted. The cavity decay rate and the spontaneous emission modify the character of tunneling in this system. In resonant case the tunneling is allowed for large tunneling splitting. In non-resonant case both the cavity decay and the spontaneous emission (a photon has certainly been emitted) lead to the exhibition that the tunneling exists and the evolution of average position oscillates with time t after a period of initial damping. $\kappa = 0$ means there is no internal degrees of freedom, the tunneling is undisturbed. $\kappa = \pi/2$ the tunneling motion is the same as the case $\kappa = \pi/4$. The entanglement of the system between the internal and the external degrees of freedom shows the entanglement sudden death. This study would shed light on the deep understanding of the tunneling as well as the disentanglement which may find applications in quantum information processing.

Acknowledgements This work is supported by NCET of M.O.E. and NSF of China under grants Nos. 60578014 and 10775023.

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